Nonlogarithmic terms in the strong field dependence of the photon propagator

1994 J. Phys. A: Math. Gen. 27 2161

(http://iopscience.iop.org/0305-4470/27/6/036)

J. Phys. A: Math. Gen. 27 (1994) 2161–2166. Printed in the UK

Non-logarithmic terms in the strong field dependence of the photon propagator

G Calucci†‡ and R Ragazzon†

† Dipartimento di Fisica Teorica dell'Università di Trieste, Trieste, I 34014 Italy
‡ INFN, Sezione di Trieste, Italy

Received 20 September 1993

Abstract. The modifications of the electromagnetic coupling in the presence of a strong external magnetic field are further investigated. The case of non-vanishing momentum transfers and the induced anisotropy of space are explicitly taken into account. We confirm the presence of directional terms showing an unusual linear dependence on the external field strength. These last terms overwhelm the expected logarithmic corrections and tend to reduce the effective coupling.

1. Purpose of the paper

The correlated e^+e^- pairs observed by the ORANGE [1] and EPOS [2] collaborations at GSI still represent a fascinating and unsolved problem. Experimental data with U+U, U+Pb, U+Th and U+Ta systems show the existence of at least three emission lines with an invariant mass of about 2 MeV and a very narrow width (~30 keV). Conventional explanations of correlated electron–positron pairs may be of two kinds. We could resort to completely known dynamics, such as a nuclear internal pair conversion, or we could assume the decay of a new neutral particle. The first process is inconsistent with the actual data, while the second one is ruled out by a number of precision tests ($g-2$ measurements, low-energy Bhabha scattering, beam dump experiments) [3]. An alternative approach to the problem has been suggested in [4, 5]. Although these works try to accommodate the main features of the data within QED, the theory is applied to extreme conditions where the perturbative treatment is not allowed. More precisely, the anomalous GSI peaks would result from the decay of bound e^+e^- states in a non-perturbative confining phase of QED with a new mass scale associated with it. Studies both on the lattices and in the continuum support the assertion that a large coupling constant, of order one, is required in order to form the new phase [6–9]. Obviously, the value $\alpha\sim\frac{1}{137}$ is well below the critical coupling involved in the proposed scenario. However, the fine structure constant can be regarded as an effective coupling constant $\alpha_{\rm eff}$ which receives correction in the presence of external fields. The hope is that an appropriate choice of the strong background fields may drive $\alpha_{\rm eff}$ from the perturbative to the strong coupling regime $\alpha_{\rm eff}\sim 1$, where the phase transition is likely to occur.

0305-4470/94/062161+06$19.50

The running of α_{eff} in a constant background field has been studied in [10] using the Schwinger's 'proper time' formalism [11] and it was soon found that the effective fine structure constant exhibits a negligible logarithmic increase, so tiny that it cannot corroborate the new phase hypothesis. Similar conclusions and suggestions can also be found in [12–14]. However, we think that the problem still deserves some clarification. In particular, in a previous paper [15] it was shown that the presence of preferred directions introduced by the background fields plays a very important role. More precisely, new terms are produced that show a linear dependence on the external field strength. The aim of this paper is to extend the long wavelength analysis of [15] to configurations where the momentum transfers are not vanishingly small. Since, in a strong electric field, it is possible to have vacuum instabilities and a complex effective coupling, we take the external field to be purely magnetic. The results obtained are in full accordance with the previous ones.

2. Quantitative estimates

The long-wavelength treatment of [15] led to the result that the dielectric permeability tensor has two eigenvalues

$$\lambda_{\perp}=1-\frac{\alpha}{6\pi}\ln(\alpha B^2/m^4) \qquad \lambda_{\parallel}=1-\frac{\alpha}{6\pi}\ln(\alpha B^2/m^4)+\frac{\alpha}{3\pi}\frac{eB}{m^2} \tag{1}$$

with eigenvectors perpendicular and parallel to $\mathbf{B}$ respectively (see in particular (38) of the quoted reference). Such eigenvalues, in turn, correspond to the effective coupling constants $\alpha_{\perp}=\alpha/\lambda_{\perp}$ and $\alpha_{\parallel}=\alpha/\lambda_{\parallel}$. The most striking feature of (1) in a very strong magnetic field is the presence of the term $(\alpha/3\pi)$ (eB/m^2) which tends to reduce the electromagnetic couplings. The aim of this section is to confirm the presence of such an effect via a computation of wider validity, which goes beyond the approximation of small exchanged momenta. More precisely, one has to investigate the strong field dependence of $\omega_{\mu\nu}(k, \mathbf{B})$ the one-loop vacuum polarization tensor to all orders in the external field $\mathbf{B}$. This problem can be dealt with by employing some useful results obtained many years ago in a different context [16, 20]. Since our main interest is to explore the strong field limit of $\omega_{\mu\nu}(k, \mathbf{B})$ we start from very general expression but we take as soon as possible the large B limit, gaining thus relevant simplifications and a deeper insight into the physical origin of the non-logarithmic terms. As in [17], our starting point is the Green function $S_F(x, y; \mathbf{B})$ which represents the exact electron propagator in a constant and homogeneous magnetic field. A rather compact form for $S_F(x, y; \mathbf{B})$ is known from the old paper by Schwinger [11] and it can be written in the form [17].

$$S_F(x, y; \mathbf{B})=\Phi(x, y)G(x-y) \tag{2a}$$

where

$$\Phi(x, y)=\exp\left[-ie\int_x^y A(\zeta)\,d\zeta\right] \tag{2b}$$

and the Fourier transforms of $G(x-y)$ can be given the following integral representation:

$$\mathscr{G}(p)=\int_0^\infty \mathrm{d}s \exp\left[-s\left(m^2+p_\parallel^2+\frac{\tanh z}{z}p_\perp^2\right)\right]$$

$$\times\left[(1+\sigma_3 \tanh z)\,(m-\gamma p_\parallel)-\frac{1}{\cosh^2 z}\gamma p_\perp\right] \tag{2c}$$

with

$$p_\parallel=(p_3, p_4) \qquad p_\perp=(p_1, p_2) \qquad z=esB \qquad \{\gamma_\mu, \gamma_\nu\}=-2\delta_{\mu\nu} \tag{2d}$$

(as usual the magnetic field is along the x_3 axis). In the fermion loop involved in the calculation of $\omega_{\mu\nu}$ the phase factors drop out and we are left with the simpler expression

$$\omega_{\mu\nu}(k)=e^2\int\frac{\mathrm{d}^4p}{(2\pi)^4}\,\mathrm{tr}[\gamma_\mu\,\mathscr{G}(p)\gamma_\nu\,\mathscr{G}(p-k)]. \tag{3}$$

The function $\mathscr{G}(p)$ is now approximated with a more compact form $\mathscr{G}_a(p)$ which holds when $eB/m^2\gg 1$. For $B\to\infty$ (and a fixed s), the integrand appearing in (2c) has a simple and regular limit, obtained by setting $\tanh(esB)\sim 1$ and $1/\cosh^2(esB)\sim 0$. These approximations are not reliable for $esB\lesssim 1$, that is $s\lesssim 1/eB$, however, this range shrinks to a point as $eB/m^2\to\infty$. Thus, noting that the integrand admits an upper bound independent of B, we can safely neglect the contribution coming from the dangerous region $s\lesssim 1/eB$ and we can readily integrate over s. As a result, the propagation function $\mathscr{G}(p)$ is cast into the form

$$\mathscr{G}_a(p)=\exp[-p_\perp^2/eB]\,(1+\sigma_3)\,\frac{m-\gamma p_\parallel}{m^2+p_\parallel^2}. \tag{4}$$

The next step is the insertion of (4) into (3). The integration over the transverse and longitudinal components of p are completely decoupled. The former is a simple Gaussian one, the latter requires the preliminary evaluation of the trace

$$T_{\mu\nu}=\mathrm{tr}[\gamma_\mu(1+\sigma_3)\,(m-\gamma p_\parallel)\gamma_\nu(1+\sigma_3)\,(m-\gamma q_\parallel)] \qquad q=p-k \tag{5}$$

which is much simplified if we use $(1+\sigma_3)\gamma_\perp(1+\sigma_3)=0$. In this way we obtain

$$T_{\mu\nu}=8[-m^2\delta^\parallel_{\mu\nu}+p^\parallel_\mu q^\parallel_\nu+q^\parallel_\mu p^\parallel_\nu-\delta^\parallel_{\mu\nu}q^\parallel p^\parallel]. \tag{6}$$

The notation implies that whenever μ or ν take a 'transverse' value, i.e. 1, 2, the tensor vanishes. The integration over the longitudinal variables may be performed by means of the Feynman parameter λ and we find, as intermediate step, the representation

$$\omega_{\mu\nu}(k, \mathbf{B})=\frac{\alpha}{8\pi^2}eB\exp[-k_\perp^2/2eB]\int_0^1\mathrm{d}\lambda\int\mathrm{d}^2p_\parallel\,\frac{T_{\mu\nu}}{[m^2+(p-\lambda k)_\parallel^2+\lambda(1-\lambda)k_\parallel^2]^2}. \tag{7}$$

This expression requires a regularization because the integral is logarithmically divergent. A gauge-invariant regularization simply removes a term independent of the fermionic mass; the integration over $p_\parallel$ is then easily carried out and gives

$$\omega_{\mu\nu}(k, \mathbf{B}) = 2\frac{\alpha}{\pi} eB\,(k_\mu^\parallel k_\nu^\parallel - \delta_{\mu\nu}^\parallel k_\parallel^2)\exp[-k_\perp^2/2eB]\int_0^1 \mathrm{d}\lambda\,\frac{\lambda(1-\lambda)}{m^2+\lambda(1-\lambda)k_\parallel^2}. \tag{8}$$

Further integration can also be performed yielding

$$\omega_{\mu\nu}(k, \mathbf{B}) = \frac{\alpha}{2\pi}(k_\mu^\parallel k_\nu^\parallel - \delta_{\mu\nu}^\parallel k_\parallel^2)\frac{eB}{m^2}\exp[-k_\perp^2/2eB]$$

$$\times\left[\frac{1}{u} - \frac{1}{2u^2}\frac{1}{(1+(1/u))^{1/2}}\ln\frac{(1+(1/u))^{1/2}+1}{(1+(1/u))^{1/2}-1}\right] \qquad u = \frac{k_\parallel^2}{4m^2}. \tag{9}$$

In the limit of small k, equations (8) and (9) give

$$\omega_{\mu\nu}(k, \mathbf{B}) \cong \frac{\alpha}{3\pi}\frac{eB}{m^2}(k_\mu^\parallel k_\nu^\parallel - \delta_{\mu\nu}^\parallel k_\parallel^2) \tag{10}$$

and reproduce therefore the leading term of (1). The origin of the linear B dependence shows up very clearly. It stems from the Gaussian distribution of the transverse fermionic momenta. In turn this shape describes the confinement of the charged particle motion in directions perpendicular to $\mathbf{B}$. The typical confinement length is of order $1/\sqrt{eB}$, as expected from very simple arguments. Since some previous investigations [10, 12–14] have reported logarithmic correction only, some words of comment may be appropriate. First, it is clear that the presence of the external field $\mathbf{B}$ breaks the isotropy of the problem. As a consequence, the dielectric permeability tensor ε_{ij} has the form

$$\varepsilon_{ij} = (1+C_1)\delta_{ij} + C_2\,n_i n_j$$

$\mathbf{n}$ being the unit vector in the direction of $\mathbf{B}$. The eigenvectors of ε_{ij} are $\lambda_\parallel = 1+C_1+C_2$, $\lambda_\perp = C_1$. A comparison with (1) shows that

$$C_1 = -\frac{\alpha}{6\pi}\ln(\alpha B^2/m^4) \qquad C_2 = \frac{\alpha}{3\pi}\frac{eB}{m^2}.$$

Therefore, we realize that the linear B dependence is missing whenever one neglects the non-trivial structure of the tensor ε_{ij}. Further consideration on this point can be found in [15].

It may be useful to give an interpretation of (4) in terms of intermediate electronic states. From (2) we have, momentum space:

$$\tilde{S}_F(p, q) = \frac{1}{\sqrt{eB}}\delta^2(p_\parallel - q_\parallel)\delta(p_2 - q_2)(1+\sigma_3)\frac{m - \gamma p_\parallel}{p_\parallel^2 + m^2}$$

$$\times\exp[-p_1^2/2eB + \mathrm{i}p_1p_2/eB]\exp[-q_1^2/2eB + \mathrm{i}q_1q_2/eB] \tag{11}$$

(in the particular gauge $A_1 = A_3 = A_4 = 0$, $A_2 = Bx_3$). Going back to the Minkowski metric, the expression shows poles for $p_0^2 = p_3^2 + m^2$, whose position is independent of B. Both $(1+\sigma_3)/2$ and $(m - \gamma p_\parallel)/2m$ are projectors (on the pole position) and they commute among themselves. It appears therefore evident that the residuum of $\tilde{S}_F$ decomposes into the product of the wavefunctions corresponding to the lowest level of a charged spinor in a static and uniform magnetic field. So we can conclude that the limit leading to (4) has the effect of pushing all the electronic Landau levels to infinity keeping only the the first one which is, in fact, independent of the magnetic field. We recall that an analysis of the vacuum polarization in terms of Landau levels has been performed in studying the generation of longitudinal modes for the photon propagation in external magnetic fields [20].

3. Concluding summary

In this work we have discussed the photon propagator in the presence of a strong and constant magnetic field. In contrast with the more common belief we have found that the corrections to the photon propagator cannot be absorbed into a logarithmically increasing effective coupling constant. Actually, as already shown by the long wavelength treatment of [15], the strong field dependence of the vacuum polarization tensor turns out to be dominated by non-isotropic terms which gives a sizeable reduction of the electromagnetic couplings. The limiting behaviour of $\omega_{\mu\nu}(k, \mathbf{B})$ has been derived in a simple way, starting from an approximated form of the exact electron propagator in a constant and homogeneous magnetic field $\mathbf{B}$. This method has the advantage of giving some hint on the origin of the unusual B dependence. In the literature, the strong field corrections to the effective fine structure constant $\alpha_{\rm eff}$ have been discussed in connection with the anomalous electron–positron pair production observed in heavy-ion collision at GSI (the so called e^+e^- puzzle or Darmstadt effect). A strong coupling regime $\alpha_{\rm eff} \sim 1$ could induce a new phase of QED which would actually explain the main features of the anomalous e^+e^- peaks. Some evidence against this mechanism has been provided in [10] where a negligible logarithmic increase of $\alpha_{\rm eff}$ is predicted. Although our results differ to some extent from those of [10] we have to draw the same conclusions about the new QED phase, since the unexpected terms appearing in the vacuum polarization tensor do not describe an increase of the electromagnetic couplings. Obviously, the role played by non-homogeneous or even time-dependent background fields is still an open and hard question [21]. Perhaps this problem is less terrible than it seems at the first sight, as suggested by the evidence that the strong field vacuum polarization can be reasonably described through consideration of only one intermediate state.

References

[1] Koenig W *et al* 1989 *Phys. Lett.* **218B** 12
[2] Salabura P *et al* 1990 *Phys. Lett.* **245B** 153
[3] Geiger K *et al* 1989 *Test of Fundamental Laws in Physics* Series: Moriond Workshops (ed O Fackler and J Tran Thanh Van) (Gif-sur-Yvette: Editions Frontières)
[4] Caldi D G and Chodos A 1987 *Phys. Rev.* D **36** 2876
[5] Ng Y J and Kikuchi Y 1987 *Phys. Rev.* D **36** 2880
[6] Guth A 1980 *Phys. Rev.* D **21** 2291
[7] Dagotto E and Kogut J 1987 *Phys. Rev. Lett.* **59** 617
[8] Dagotto E and Kogut J 1988 *Nucl. Phys.* B **295** [FS21] 123
[9] Dagotto E, Kogut J and Kocic A 1988 *Phys. Rev. Lett.* **60** 772
[10] Chodos A, Owen D and Sommerfield C 1988 *Phys. Lett.* **212B** 491
[11] Schwinger J 1951 *Phys. Rev.* **82** 664
[12] Matinyan S G and Savvidy G K 1978 *Nucl. Phys.* B **134** 539
[13] Peccei, R D, Solà J and Wetterich C 1988 *Phys. Rev.* D **37** 2492
[14] Ng Y J and Müller B 1989 *Test of Fundamental Laws in Physics* Series: Moriond Workshops (ed O Fackler and J Tran Thanh Van) (Gif-sur-Yvette: Editions Frontières)

[15] Ragazzon R 1992 *J. Phys. A: Math. Gen.* **25** 2997
[16] Tsai W-y and Erber T 1974 *Phys. Rev.* D **10** 492
[17] Tsai W-y 1974 *Phys. Rev.* D **10** 2699
[18] Tsai W-y and Erber T 1975 *Phys. Rev.* D **12** 1132
[19] Tsai W-y and Erber T 1976 *Acta Phys. Austriaca* **45** 245
[20] Cover R A and Kalman G 1974 *Phys. Rev. Lett.* **33** 1113
[21] Caldi D G and Vafaeisefat S 1991 *Preprint* BNL 46384